MATH 112A Review: Laplace and Polar Coordinates

1. Let $f(x,y) = xy + x^2y$. What is Δf?

   Solution: We have that $f_x = y + 2xy$ and $f_y = x + x^2$. Thus, $f_{xx} = 2y$ and $f_{yy} = 0$. Hence, $\Delta f = 2y$.

2. Let $f(x,y) = e^{\sqrt{x^2+y^2}}$. What is Δf in polar coordinates?

   Solution: Let $g(r,\theta) = f(r\cos x, r\sin\theta)$. Then,

   $$g(r,\theta) = e^{\sqrt{r^2\cos^2\theta + r^2\sin^2\theta}} = e^r.$$

   Hence, $g_{\theta\theta} = 0$ and $g_r = e^r$. Thus, $\Delta f = \frac{1}{r}\frac{\partial}{\partial r}(rg_r) + 0 = \frac{1}{r}\frac{\partial}{\partial r}(re^r) = \frac{1}{r}e^r + e^r$ in polar coordinates.

3. Let $f(x,y,z) = xyz$. What is Δf?

   Solution: We have that $f_x = yz$, $f_y = xz$, and $f_z = xy$. Hence, $f_{xx} = 0$, $f_{yy} = 0$, and $f_{zz} = 0$. Hence, $\Delta f = 0$.